Calculators are not allowed

Answer the following questions:

1. (a) (3 points) Evaluate the following limit (if it exists):

$$\lim_{x \to 0} \frac{\cos x - 1}{x \sin x}.$$

   (b) (3 points) State Rolle's theorem.

2. Let $f(x) = (x^2 - x - 2)^{\frac{2}{3}}$.

   (a) (2 points) Discuss the continuity of f.

   (b) (4 points) Find the points at which the graph of f has a cusp (if any).

3. (6 points) If a box with a square base and an open top is to have a surface area $300 \ cm^2$, find its maximum volume.

4. (4+4 points) Evaluate the following integrals,

   (a) $\displaystyle\int \left(1 + \frac{1}{x}\right)^{-3} \frac{1}{x^2} dx,$

   (b) $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\sin x + 1} \cos x\, dx.$

5. (a) (3 points) Show that

$$\int \sin^3 x\, dx = \frac{1}{3}\cos^3 x - \cos x + C,$$

   where C is an arbitrary constant.

   (b) (3 points) If the average value of $f(x) = x^3 + bx - 2$ on the interval $[0, 2]$ is 4, find the constant b.

6. (6 points) The region bounded by the graphs of the curves $y = x^2 - 3$ and $y = 5 - x^2$ is revolved about the line $x = 6$. Find the volume of the resulting solid.

7. (a) (6 points) Let $f(x) = \displaystyle\int_0^x \sqrt{t^4 + 2t^2}\, dt$. Find tha arc length of the graph of the equation $y = f(x)$ from $A(0, f(0))$ to $B(1, f(1))$.

   (b) (6 points) Find the area of the region bounded by the graphs of $y = x^2$ and $y = x^3$.

Good Luck